Equations of Motion

ASSUME ALL DIGITS GIVEN ARE SIGNIFICANT

1) A car travelling at 50 km/h East decelerates at 6.05m/s^2 West until it comes to rest. (Convert the 50 km/h into m/s)

   a) How long does it take the car to come to rest?

$\vec{v}_1 = 50 km/h = 13.9 m/s$
$\vec{a} = -6.05 m/s^2$
$\vec{v}_2 = 0 m/s$
$\Delta t = ?$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

$\Delta t = \frac{\Delta \vec{v}}{\vec{a}}$

$\Delta t = \frac{0 m/s - 13.9 m/s}{-6.05 m/s^2}$

$\Delta t \approx 2.3 s$

   b) How far does the car travel in this time period?

$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d}$

$\vec{v}_2^{\,2} - \vec{v}_1^{\,2} = 2\vec{a}\Delta d$

$\Delta\vec{d} = \frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\vec{a}}$

$\Delta\vec{d} = \frac{0^2 - (13.9 m/s)^2}{2(-6.05 m/s^2)}$

$\Delta\vec{d} \approx 16.0 m$

2) A rocket ship accelerates at 6g (g = 9.80 m/s^2), upward, during lift off. What is the ships velocity after 6 seconds? 300 seconds?

$\vec{a} = 6(9.80 m/s^2)$
$V_1 = 0 m/s$
$V_2 = ?$
$t = 6 s$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$ ($\vec{v}_1$ crossed out, 0)

$\vec{v}_2 = \vec{a}\Delta t$

$\vec{v}_2 = 6(9.80 m/s^2)(6 s)$

$\vec{v}_2 \approx 360 m/s$

$\vec{v}_2 = 6(9.80 m/s^2)(300)$

$\vec{v}_2 \approx 17\,600 m/s$

3) A bicyclist accelerates at .45 m/s^2 north for 18.0s and reaches a velocity of 19.8 m/s north.

   a) What was the initial velocity of the bicycle?

$\vec{a} = 0.45 m/s^2$
$\Delta t = 18.0 s$
$\vec{v}_2 = 19.8 m/s$
$\vec{v}_1 = ?$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$\vec{v}_2 - \vec{a}\Delta t = \vec{v}_1$

$\vec{v}_1 = 19.8 m/s - 0.45 m/s^2 (18.0 s)$

$\vec{v}_1 \approx 11.7 m/s$

b) How far did the bicycle travel while accelerating?

$\Delta\vec{d} = ?$

$\Delta\vec{d} = \vec{v}_2 \Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$

$\Delta\vec{d} = 19.8\ m/s(18\ s) - \frac{1}{2}(0.45\ m/s^2)(18\ s)^2$

$\Delta\vec{d} \approx 280\ m$

4) In a drag race a car travels in one direction down a .40 km racetrack. Some cars reach speeds of 550 km/h at the end of the race.

a) What is the average acceleration of these cars during the race?

$\vec{v}_1 = 0\ km/h$
$\vec{v}_2 = 550\ km/h$
$\Delta d = 0.40\ km$
$\vec{a} = ?$

$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta d$

$\frac{\vec{v}_2^{\,2}}{2\Delta d} = \vec{a}$

$\vec{a} = \frac{(550\ km/h)^2}{2(0.40\ km)}$

$\vec{a} \approx 380\,000\ km/h^2$

c) How much time would the car take?

$\Delta t = ?$

$\Delta\vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$

$\frac{2\Delta\vec{d}}{v_2 + v_1} = \Delta t$

$\Delta t \approx \frac{2(0.40\ km)}{550\ km/h}$

$\Delta t \approx 0.0015\ h$

$\Delta t \approx 5.2\ s$

5) A sprinter completes a 200m race in 29.9s.

a) What was the runner's average speed?

$\Delta d = 200\ m$
$\Delta t = 29.9\ s$
$\vec{v}_{av} = ?$

$\vec{v}_{av} = \frac{\Delta d}{\Delta t}$

$v_{av} = \frac{200\ m}{29.9\ s}$

$v_{av} \approx 6.69\ m/s$

b) What was the average acceleration during the race?

$\Delta\vec{d} = 200\ m$
$\Delta t = 29.9\ s$
$v_1 = 0\ m/s$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$

$\vec{a} = \frac{2\Delta d}{\Delta t^2}$

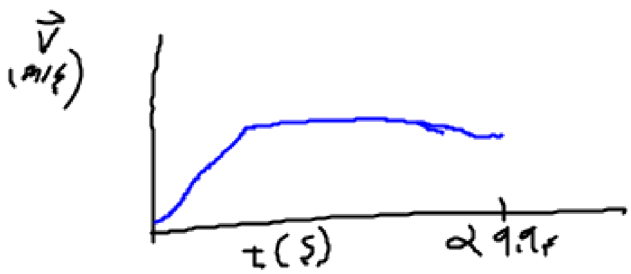
$\vec{a} = \frac{2(200\ m)}{(29.9\ s)^2}$

$\vec{a} \approx 0.4\ m/s^2$

c) Why is the average acceleration not the actual acceleration of the runner?

The runner does not accelerate for the whole race → they eventually reach a top speed.

d) Sketch a graph of what you'd expect the runner's velocity to book like as a function of time.



6) During a head-on collision a vehicles airbag increases the time for a person's head to stop moving from .10 s to .30s. If the vehicle was travelling at 28m/s, by what factor would the airbags decrease the acceleration of the people involved in the crash?

.10s

$\vec{a} = \frac{\Delta v}{\Delta t}$

$\vec{a} = \frac{-28 m/s}{0.10 s}$

$\vec{a} = -280 m/s^2$

.30s

$\vec{a} = \frac{-28 m/s}{0.30 s}$

$\vec{a} \approx -93 m/s^2$

Compare:

$\frac{-280 m/s^2}{-93 m/s^2}$

≈ a factor of 3

7) A drag racer travelling at 240km/h on a straight track deploys a parachute that provides a uniform acceleration that slows the racer to 20km/h in 12 s.

a) What is the racer's acceleration?

$v_1 = 240 km/h$

$v_2 = 20 km/h$

$\Delta t = 12 s$

$\vec{a} = ?$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

$\vec{a} = \frac{20 km/h - 240 km/h}{12 s}$

$\vec{a} \approx -18 km/h/s$

b) How far does the racer travel in the 12s interval?

$\vec{\Delta d} = ?$

$\Delta t = 12 s \approx 0.0033 h$

$\Delta \vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$

$\vec{\Delta d} = \frac{1}{2}(240 km/h + 20 km/h)(0.0033 h)$

$\Delta d \approx 0.43 km$

8) An astronaut drops a feather from 1.2 m above the surface of the moon. If the acceleration due to gravity is 1.62 m/s^2 down, how long does it take the feather to hit the surface? [1.2 s]

9) A world class sprinter can reach a top speed of about 13m/s in 3.0s. a) What is the acceleration of the sprinter? b) How much distance does the sprinter travel while accelerating? c) If the sprinter could maintain this speed for the entire 100m dash what would her time be? [4.33m/s^2; 19.5m; 9.19s]

a) $\vec{a} = \frac{13 m/s}{3.0 s}$

$\vec{a} \simeq 4.3 m/s^2$

b) $\Delta\vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$

$\Delta\vec{d} = \frac{1}{2}(13 m/s + 0 m/s)(3.0 s)$

$\Delta\vec{d} \simeq 19.5 m$

c) $\Delta t = \frac{\Delta\vec{d}}{\Delta\vec{v}}$

$\Delta t = \frac{100 m}{13 m/s}$

$\Delta t \simeq 7.7 s$

10) A car travelling on icy roads at 30m/s applies the brakes and slides 100m until reaching a speed of 19m/s. The out of control car then slams into a parked truck. The truck and car slide a further 6.5m before they both come to rest.

a) What acceleration does the car experience while sliding before striking the truck?

$\vec{v}_1 = 30 m/s$

$\vec{v}_2 = 19 m/s$

$\Delta d = 100 m$

$\vec{a} = ?$

$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta d$

$\frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\Delta d} = \vec{a}$

$\vec{a} = \frac{(19 m/s)^2 - (30 m/s)^2}{2(100 m)}$

$\vec{a} \simeq -2.7 m/s^2$

b) What acceleration is felt in the car during the collision? [-2.7m/s^2; -27.8m/s^2]

$\vec{v}_1 = 19 m/s$

$\vec{v}_2 = 0 m/s$

$\Delta d = 6.5 m$

$\vec{a} = ?$

$\vec{a} = \frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\Delta d}$

$\vec{a} = \frac{(0 m/s)^2 - (19 m/s)^2}{2(6.5 m)}$

$\vec{a} \simeq -28 m/s^2$

11) An armored tank travelling at 3m/s accelerates at 9.80m/s^2 for a distance of 20m.

a) What is the tanks velocity after the 20m?

b) For how much time is the tank accelerating? [20.0m/s; 1.73s]

12) A car with a velocity of 22m/s is accelerated uniformly at the rate of 1.6m/s^2 for 6.8s. What is its final velocity? [32.9m/s]

13) A supersonic jet flying at 145m/s is accelerated unifomly at the rate of 23.1m/s^2 for 20.0s.

a) What is its final velocity?

$\vec{v}_1 = 145 m/s$

$\vec{a} = 23.1 m/s^2$

$\Delta t = 20.0 s$

$\vec{v}_2 = ?$

$\vec{v}_2 = \vec{v}_1 + a\Delta t$

$\vec{v}_2 = 145 m/s + 23.1 m/s^2 (20.0 s)$

$\vec{v}_2 \approx 607 m/s$

b) Using the speed of sound in air as 331 m/s, how many times the speed of sound is the plane's final speed? [607m/s; 1.83]

$\frac{607 m/s}{331 m/s} \approx \boxed{1.83}$

14) Determine the displacement of a plane that is uniformly accelerated from 66m/s to 88m/s in 12s. [924m]

15) How far does a plane fly in 15s while its velocity is changing from 145m/s to 75m/s at a uniform rate of acceleration? [1650m]

$\Delta t = 15s$
$\vec{v}_1 = 145 m/s$
$\vec{v}_2 = 75 m/s$
$\Delta \vec{d} = ?$

$\Delta \vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$

$\Delta d = \frac{1}{2}(75 m/s + 145 m/s)(15 s)$

$\Delta d = 1650 m$

16) A car moves at 12m/s and coasts up a hill with a uniform acceleration of -1.6m/s^2.

a) How far has it traveled after 6.0s?

b) Where is it at 9.0s? [43.2m; 43.2m]

17) A plane travels 500m while being accelerated uniformly from rest at the rate of 5.0m/s^2. What final velocity does it attain? [70.7m/s]

18) A pilot stops a plane in 484 m using a constant acceleration of -8.0 m/s. How fast was the plane moving before braking began?

19) A race car can be slowed with a constant acceleration of $-11 m/s^2$.

a) If the car is going 55m/s, how many meters will it take to stop?

b) How many meters will it take to stop a car going twice as fast? [138m; 550m]

20) An engineer must design a runway to accommodate airplanes that must reach a ground velocity of 61m/s before they can take off. These planes are capable of being accelerated uniformly at the rate of $2.5 m/s^2$.

a) How long will it take the planes to reach takeoff speed?

$\Delta t = ?$

$\vec{v}_1 = 0$

$\vec{v}_2 = 61 m/s$

$\vec{a} = 2.5 m/s^2$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

$\Delta t = \frac{\Delta \vec{v}}{\vec{a}}$

$\Delta t = \frac{61 m/s - 0 m/s}{2.5 m/s^2}$

$\Delta t \approx 24 s$

b) What must be the minimum length of the runway? [24.4s; 744m]

$\Delta d = ?$

$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2a\Delta\vec{d}$ (with $\vec{v}_1^{\,2}$ crossed out, $= 0$)

$\frac{\vec{v}_2^{\,2}}{2a} = \Delta d$

$\frac{(61 m/s)^2}{2(2.5 m/s^2)} = \Delta \vec{d}$

$740 m \approx \Delta \vec{d}$

21) Engineers are developing new types of guns that might someday be used to launch satellites as if they were bullets. One such gun can give a small object a velocity of 3.5km/s, moving it through only 2.0cm.

a) What acceleration does the gun give this object?

b) Over what time interval does the acceleration take place? [$3.06 \times 10^{8} m/s^2$; $1.14 \times 10^{-5} s$]

22) Highway safety engineers build soft barriers so that cars hitting them will slow down at a safe rate. A person wearing a seat belt can withstand an acceleration of $-3.0 \times 10^{2} m/s^2$. How thick should barriers be to safely stop a car that hits a barrier at 110km/h? [1.56m]

23) A baseball pitcher throws a fastball at a speed of 44m/s. The acceleration occurs as the pitcher holds the ball in his hand and moves it through an almost straight-line distance of 3.5m. Calculate the acceleration, assuming it is uniform. Compare this acceleration to the acceleration due to gravity, $9.80 m/s^2$. [$277 m/s^2$; 28.2g's]

24) Rocket-powered sleds are used to test the responses of humans to acceleration. Starting from rest, one sled can reach a speed of 444m/s in 1.80s and can be brought to a stop again in 2.15s.

a) Calculate the acceleration of the sled when starting, and compare it to the magnitude of the acceleration due to gravity, 9.80m/s^2.

$\vec{v}_1 = 0$
$\vec{v}_2 = 444 m/s$
$\Delta t = 1.80 s$
$\vec{a} = ?$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

$\vec{a} = \frac{444 m/s - 0 m/s}{1.80 s}$

$\vec{a} \approx 247 m/s^2$

$\frac{247 m/s^2}{9.80 m/s^2} = \boxed{25.2 g's}$

b) Find the acceleration of the sled when braking and compare it to the magnitude of the acceleration due to gravity. [247m/s^2 = 25.2 g's; -207m/s^2 = -21.1 g's]

$\vec{v}_1 = 444 m/s$
$\vec{v}_2 = 0 m/s$
$\Delta t = 2.15 s$
$\vec{a} = ?$

$\vec{a} = \frac{\Delta v}{\Delta t}$

$\vec{a} = \frac{0 m/s - 444 m/s}{2.15 s}$

$\boxed{\vec{a} \approx -207 m/s}$

$\frac{-207 m/s}{9.80} \approx \boxed{-21.1 g's}$

25) As a traffic light turns green, a waiting car starts with a constant acceleration of 6.0m/s^2. At the instant the car begins to accelerate, a truck with a constant velocity of 21m/s passes in the next lane.

a) How far will the car travel before it overtakes the truck?

$\vec{v}_{c1} = 0 m/s$
$\vec{a}_c = 6.0 m/s^2$
$\vec{v}_t = 21 m/s$
$\Delta d_c = \Delta d_t = ?$
$\Delta t_c = \Delta t_t = \Delta t$

$\Delta \vec{d}_t = v_t \Delta t$
$\Delta \vec{d}_c = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} (\Delta t)^2$
$\Delta \vec{d}_t = \Delta d_c$
$\therefore v_t \Delta t = \frac{1}{2} a_c (\Delta t)^2$
$\frac{2 v_t}{a_c} = \Delta t$

$\Delta t = \frac{2(21 m/s)}{6.0 m/s^2}$
$\Delta t \approx 7.0 s$
$\Delta d_c = \frac{1}{2}(6.0 m/s)(7.0 s)^2$
$\boxed{\Delta d_c \approx 150 m}$

b) How fast will the car be travelling when it overtakes the truck? [147m; 42.0,m/s]

$\vec{v}_{c2} = ?$

$\vec{v}_{2c} = \vec{v}_{1c} + \vec{a}_c \Delta t$

$\vec{v}_{2c} = (6.0 m/s^2)(7 s)$

$\boxed{\vec{v}_{2c} \approx 42 m/s}$

26) A jet aircraft being launched from an aircraft carrier is accelerated from rest along a 94m track for 2.50s.

a) What is the acceleration of the aircraft, assuming it is constant?

b) What is the launch speed of the jet? [30m/s^2; 75m/s]

27) A rifle bullet with a muzzle speed of 330m/s is fired directly into a special, dense material that stops the bullet in 30.0cm. Assuming the bullet's deceleration to be constant, what is its magnitude? [1.8 x 10^5 m/s^2]

28) A driver of a car going 90 km/h suddenly sees the lights of a barrier 40.0 m ahead. It takes the driver 0.75s before she applies the brakes, and the average acceleration during breaking is -10.0 m/s^2.

   a. Determine if the car hits the barrier. [Yes]

b. What is the maximum speed at which the car could be moving and not hit the barrier 40 m ahead? Assume the acceleration rate does not change. [22 m/s]

29) The speed limit in a school zone is 40.0km/h. A driver travelling at this speed sees a child run onto the road 13m in front of his car. He applies the brakes, and the car decelerates at a uniform rate of 8.0m/s^2.

a. If the drivers reaction time is 0.25s, will the car stop before hitting the child and how far will the car take to stop? [yes; 10.5m]

$\vec{v}_1 = 40.0 km/h = 11.1 m/s$

$a_c = -8.0 m/s^2$

$\Delta d_K = 13 m$

$\Delta t_R = 0.25 s$

//Before stopping

$\Delta \vec{d} = \vec{v} \Delta t$

$\Delta d_R = 11.1 m/s (0.25 s)$

$\Delta d_R = 2.78 m$

//distance to stop

$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d}$ ($\vec{v}_2 = 0$)

$\frac{-v_1^2}{2\vec{a}} = \Delta\vec{d}$

$\Delta\vec{d}_s = \frac{-(11.1 m/s)^2}{2(-8.0 m/s^2)}$

$\Delta\vec{d}_s \approx 7.7 m$

//Total distance:

$\Delta\vec{d} = \Delta d_R + \Delta d_s$

$\Delta\vec{d} = 2.78 m + 7.7 m$

$\Delta d \approx 10 m$

Yes :)

b. Repeat the question above using a reaction time of 0.50s instead of 0.25s. [no; 13.3m]

$\Delta d_R = ?$

$\Delta d_s = 7.7 m$

$\Delta t_R = 0.50 s$

$\Delta d_R = \Delta \vec{v}_c \Delta t_R$

$\Delta\vec{d}_R = (11.1 m/s)(0.50 s)$

$\Delta\vec{d}_r \approx 5.6 m$

$\Delta\vec{d} = 5.6 m + 7.7 m$

$\Delta\vec{d} = 13.3 m$

NO! :(